

# A proposal of a fine tuning free formulation of 4d $\mathcal{N} = 4$ super Yang-Mills

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## Abstract

Recently, a nonperturbative formulation of 4d  $\mathcal{N} = 4$  super Yang-Mills theory which does not require fine tuning at least to all order in perturbation theory has been proposed by combining two-dimensional lattice and matrix model techniques. In this paper we provide an analogous model by utilizing deconstruction approach of Kaplan et al. Two-dimensional lattice with a plane wave deformation is deconstructed from a matrix model and two additional dimensions emerge through the Myers effect. In other words we construct a D1-brane theory from which a D3-brane theory comes out. The action is much simpler than the previous formulation and hence numerical study, which enables us to test the  $AdS_5/CFT_4$  duality at fully nonperturbative level, becomes much easier.

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# 1 Introduction

Supersymmetric Yang-Mills (SYM) theories play prominent roles in theoretical particle physics. Among them, maximally supersymmetric theories are of crucial importance for superstring/M theory [1, 2, 3, 4]. Given that most interesting questions can be answered only through nonperturbative study, it is important to construct theoretical frameworks for that. However, it is not a straightforward task because of the notorious difficulties of lattice supersymmetry (SUSY). So far, lattice formulations which are free from fine tunings are established only for one- and two-dimensional theories, three-dimensional maximally supersymmetric theory and 4d  $\mathcal{N} = 1$  pure SYM.<sup>2</sup> It motivated people to study *non-lattice* approaches to SYM.

For 1d theory (matrix quantum mechanics), lattice is not necessary at all thanks to the absence of UV divergence and simple momentum cutoff prescription works [8]. By using it, remarkable quantitative agreement with the gauge/gravity duality conjecture [4] has been obtained [9] (for lattice study with qualitatively consistent result, see [10]). By combining the momentum cutoff or lattice techniques with a plane wave deformation [11] and the Myers effect [12], 3d theory can be obtained as an expansion of 1d matrix model around fuzzy sphere [13]. Also, in the planar limit, 4d theory can be obtained using a novel large- $N$  reduction technique [14, 15] inspired by the Eguchi-Kawai equivalence [16].

In order to construct 4d  $\mathcal{N} = 4$  SYM at a finite rank of a gauge group, one can combine fuzzy sphere technique [13] with 2d SYM; that is, by constructing 2d SYM with the plane wave deformation using standard lattice SUSY techniques and then taking fuzzy sphere background, 4d SYM is naturally realized. Such a model is constructed in [17] by modifying Sugino's 2d lattice model [18, 19, 20], and the absence of fine tuning problem to all order in perturbation theory has been shown. Whether fine tunings are absent at nonperturbative level should be checked by numerical simulation. However note that in other models the absence is not shown even at perturbative level<sup>3</sup>.

Although this model possesses beautiful features, however, the action is rather complicated and it is not easy to put it on computer. Therefore, in this paper we construct similar, but much simpler, model by utilizing the deconstruction method (or “deconstruction/orbifolding approach”) of Kaplan et al. [22, 23, 24]<sup>4</sup>. The action is as simple as the original non-deformed model [24] and can easily be put on computer.

## 2 Basic idea

First let us remind a matrix model construction of 3d maximally supersymmetric Yang-Mills theory [13]; in short, *D2-brane theory (3d SYM) emerges from D0-brane theory (1d SYM)*. The starting point is the  $U(N)$  plane wave matrix model of Berenstein-Maldacena-Nastase [11]. Fuzzy sphere solution to this model, which is interpreted as compact fuzzy D2-branes, preserves 16 SUSY. Around  $k$ -coincident fuzzy sphere 3d  $U(k)$  theory (D2-brane theory) on noncommutative space is

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<sup>2</sup> For recent numerical studies of 4d  $\mathcal{N} = 1$  theory, see [5]. For review of orbifold method which is utilized in this paper, see [6]. Note however that a part of numerical data for super Yang-Mills theories shown in this review is problematic; see [7].

<sup>3</sup> In a class of lattice models, fine tuning is not needed at one-loop level [21]. It would be nice to study whether it is the case at higher order.

<sup>4</sup> For other constructions, see [25, 26, 27]. Relationship between various models are discussed in [28].

realized. At any fixed  $N$ , this is nothing but 1d theory and can easily be regularized. By taking the continuum limit as 1d theory, full SUSY is restored automatically, without requiring any fine tuning. *By taking continuum limit first for 1d (time) direction and then along spherical directions, 3d SYM is realized without parameter fine tuning*<sup>5</sup>. Note that for the maximally supersymmetric theory commutative limit of noncommutative space is expected to be smooth [30, 31].

To obtain 4d theory, one can start with 2d  $U(N)$  SYM with the plane wave deformation [32]; *D3-brane theory (4d SYM) is obtained from D1-brane theory (2d SYM)*<sup>6</sup>. Four-dimensional  $U(k)$  theory on two commutative and two noncommutative dimensions naturally arises around  $k$ -coincident fuzzy sphere background. By taking an appropriate large- $N$  limit, 4d  $\mathcal{N} = 4$  SYM on commutative  $\mathbb{R}^4$  is obtained [17]. (This point is further explained in § 6.)

In [17], such 2d lattice is constructed by generalizing Sugino model [18, 19, 20]. However the action is rather complicated and cannot easily be put on computer. Actually already before turning on the plane wave deformation Sugino's action is much more complicated than “deconstruction” model of Kaplan et al. [22, 23, 24]; so we repeat the program pursued in [17], this time using the deconstruction technique, and construct a simple action which is convenient for numerical simulation.

We start with a zero-dimensional maximally supersymmetric matrix model (IIB matrix model) [2], which was used as a starting point in [24]. To IIB matrix model we add “plane-wave deformation”, by utilizing the results obtained in [17]. From that we construct 2d theory following the procedure of Kaplan et al. It turns out that the procedure in [24] applies perfectly in parallel; Although we have a small number of deformation terms, essentially the same orbifolding condition can be used.

In summary, there are three basic steps:

- Add supersymmetric “plane wave” deformation to IIB matrix model. (§ 4)
- From the matrix model, generate 2d lattice through deconstruction of Kaplan et al. Then take continuum limit. (§ 5)
- From 2d SYM, generate 4d SYM through the Myers effect. (§ 6)

### 3 Brief introduction to the deconstruction

In this section we provide a short review of the deconstruction method [37]. Although we explain only two-dimensional lattice, generalizations to other dimensions are straightforward.

Let us start with bosonic 4-matrix model, which is obtained from 4d pure Yang-Mills theory through the dimensional reduction (“*mother theory*”)

$$S = -\frac{1}{4g_{0d}^2} \text{Tr}[X_I, X_J]^2, \quad (1)$$

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<sup>5</sup> Similar anisotropic continuum limit is considered also in the framework of the deconstruction in order to reduce the number of fine tunings [22].

<sup>6</sup> Another possible option is to consider four-dimensional noncommutative space out of zero-dimensional theory. For example in [33] and [34] realization by using fuzzy four-torus and fuzzy  $S^2 \times S^2$  have been discussed. However these geometries have flat or tachyonic directions [35] and it is not clear whether the geometry can be stabilized by adding soft deformations. Other geometries like fuzzy  $\mathbb{CP}^2$  might be useful [36]. Note also that the boundary condition of the fermions cannot be changed in these constructions and hence thermal properties cannot be studied.

where  $X_I (I = 1, \dots, 4)$  are  $N \times N$  hermitian matrices. We take  $N = L^2 M$ , where  $L$  and  $M$  are integers as well.  $L$  translates into the size of lattice, while  $M$  specifies the gauge group  $U(M)$ .

By using complex fields  $x \equiv X_1 + iX_2$  and  $y \equiv X_3 + iX_4$ , the action can be written as

$$S = \frac{1}{4g_{0d}^2} \text{Tr} \left\{ \frac{1}{2} |[x, \bar{x}] + [y, \bar{y}]|^2 + 2|[x, y]|^2 \right\}. \quad (2)$$

Here  $\bar{x} = X_1 - iX_2$  and  $\bar{y} = X_3 - iX_4$ .

In the deconstruction method, two-dimensional lattice (“*daughter theory*”) is obtained from the matrix model (mother) through the orbifolding. For fields  $\Pi (= x, y)$ , we introduce “orbifolding condition”

$$C_i \Pi C_i^{-1} = \omega^{r_i} \Pi, \quad \omega = e^{2\pi i/L} \quad (3)$$

where  $C_1$  and  $C_2$  are given by

$$C_1 = \Omega \otimes \mathbf{1}_L \otimes \mathbf{1}_M, \quad C_2 = \mathbf{1}_L \otimes \Omega \otimes \mathbf{1}_M, \quad (4)$$

and

$$\Omega = \text{diag}(\omega^{-1}, \omega^{-2}, \dots, \omega^{-L}) \quad (5)$$

is the clock matrix. Here we take

$$\vec{r}_x = (1, 0), \quad \vec{r}_y = (0, 1). \quad (6)$$

Then, the only non-vanishing components are  $x_{n_1, n_2, k; n_1+1, n_2, l}$  and  $y_{n_1, n_2, k; n_1, n_2+1, l}$  ( $n_1, n_2 = 1, \dots, L; k, l = 1, \dots, M$ ). We interpret  $(n_1, n_2)$  to be a label of site on  $L \times L$  periodic lattice. Also we regard  $x$  and  $y$  as link variables connecting  $(n_1, n_2)$  and  $(n_1 + 1, n_2)$ ,  $(n_1, n_2 + 1)$ , respectively. By denoting  $x_{\vec{n}, kl} \equiv x_{n_1, n_2, k; n_1+1, n_2, l}$  and  $y_{\vec{n}, kl} \equiv y_{n_1, n_2, k; n_1, n_2+1, l}$ , one obtains

$$S^{\text{lat}} = \frac{1}{4g_{0d}^2} \sum_{\vec{n}} \text{Tr} \left\{ \frac{1}{2} |x_{\vec{n}} \bar{x}_{\vec{n}} - \bar{x}_{\vec{n}-\hat{x}} x_{\vec{n}-\hat{x}} + y_{\vec{n}} \bar{y}_{\vec{n}} - \bar{y}_{\vec{n}-\hat{y}} y_{\vec{n}-\hat{y}}|^2 + 2|x_{\vec{n}} y_{\vec{n}+\hat{x}} - y_{\vec{n}} x_{\vec{n}+\hat{y}}|^2 \right\}. \quad (7)$$

Here  $\text{Tr}$  indicates the trace for  $M \times M$  matrix. By expanding it around

$$x_{\vec{n}} = \frac{1}{a} + s_{1, \vec{n}} + iA_{1, \vec{n}}, \quad y = \frac{1}{a} + s_{2, \vec{n}} + iA_{2, \vec{n}}, \quad (8)$$

and by taking  $g_{2d}^2 \equiv a^2 g_{0d}^2$ , tree-level continuum limit becomes

$$S^{\text{cont, tree}} = \frac{1}{2g_{2d}^2} \int d^2x \left( F_{12}^2 + (D_\mu s_I)^2 - [s_1, s_2]^2 \right). \quad (9)$$

Similar expansion can be performed around more generic background  $x_{\vec{n}} \bar{x}_{\vec{n}} = y_{\vec{n}} \bar{y}_{\vec{n}} = \frac{1}{a^2} \cdot \mathbf{1}$ .

When  $x_{\vec{n}} \bar{x}_{\vec{n}}$  and  $y_{\vec{n}} \bar{y}_{\vec{n}}$  deviate from  $\frac{1}{a^2} \cdot \mathbf{1}$ , the continuum limit does not agree with Yang-Mills theory; in terms of  $U_i$ , it corresponds to the situation that the background value of plaquette deviates from 1. In order to avoid such pathological situation, one adds

$$\frac{\nu^2 a^2}{8g_{0d}^2} \sum_{\vec{n}} \text{Tr} \left( \left| x_{\vec{n}} \bar{x}_{\vec{n}} - \frac{1}{a^2} \right|^2 + \left| y_{\vec{n}} \bar{y}_{\vec{n}} - \frac{1}{a^2} \right|^2 \right) \quad (10)$$

to the action. In the continuum, this amounts to adding a scalar mass term  $(\nu^2/2g_{2d}^2) \int d^2x \text{Tr}(s_1^2 + s_2^2)$ .

The advantage of the deconstruction method when we consider supersymmetric theory is [22] *if the SUSY transformation generated by a supercharge  $Q$  which relates fields of the same charge  $\vec{r}$ , the daughter theory automatically has an exact supersymmetry generated by (projected version of)  $Q$ .* This fact is obvious but rather powerful, because supersymmetric matrix model can easily be obtained just by dimensional reduction, and an appropriate choice of  $\vec{r}$  is naturally obtained from R-symmetry charge.

## 4 0d “mother” theory

Let us start with the IIB matrix model [2], which is the zero-dimensional reduction of 4d  $\mathcal{N} = 4$  SYM. It is written as

$$S_0 = \frac{1}{g_{0d}^2} \text{Tr} \left\{ -\frac{1}{4} [X^I, X^J]^2 + \frac{i}{2} \Psi^T \gamma_I [X^I, \Psi] \right\}, \quad (11)$$

where  $I = 1, \dots, 10$ , and  $\gamma_I$  ( $I = 1, \dots, 10$ ) are  $16 \times 16$  sub-sectors of 10d gamma matrices.

In order to introduce plane wave deformation, it is useful to switch to another (at first sight rather complicated) notation<sup>7</sup> by using Hermitian scalars  $X_i$  ( $i = 1, 2, 3, 4$ ),  $B_A$  ( $A = 1, 2, 3$ ) and  $C$ , complex scalars  $\phi_{\pm}$ , bosonic auxiliary fields  $H_A$ ,  $\tilde{H}_i$ , and fermionic variables  $\psi_{\pm i}$ ,  $\chi_{\pm A}$  and  $\eta_{\pm}$ . There are appropriate supercharges  $Q_{\pm}^{(0)}$  by which  $S_0$  can be written in exact form as

$$S_0 = Q_+^{(0)} Q_-^{(0)} \mathcal{F}^{(0)}, \quad (12)$$

where

$$\begin{aligned} \mathcal{F}^{(0)} = & \frac{1}{2g_{0d}^2} \text{Tr} \left\{ -i B_A \Phi_A - \frac{1}{3} \epsilon_{ABC} B_A [B_B, B_C] \right. \\ & \left. - \psi_{+i} \psi_{-i} - \chi_{+A} \chi_{-A} - \frac{1}{4} \eta_+ \eta_- \right\}, \end{aligned} \quad (13)$$

and  $\Phi_1 = 2(-i[X_1, X_3] - i[X_2, X_4])$ ,  $\Phi_2 = 2(-i[X_1, X_4] + i[X_2, X_3])$ ,  $\Phi_3 = 2(-i[X_1, X_2] + i[X_3, X_4])$ . Supercharges  $Q_{\pm}^{(0)}$  transform fields as

$$\begin{aligned} Q_{\pm}^{(0)} X_i &= \psi_{\pm i}, \quad Q_{\pm}^{(0)} \psi_{\pm i} = \mp [X_i, \phi_{\pm}], \\ Q_{\mp}^{(0)} \psi_{\pm i} &= -\frac{1}{2} [X_i, C] \mp \tilde{H}_i, \\ Q_{\pm}^{(0)} \tilde{H}_i &= [\phi_{\pm}, \psi_{\mp i}] \mp \frac{1}{2} [C, \psi_{\pm i}] \pm \frac{1}{2} [X_i, \eta_{\pm}], \\ Q_{\pm}^{(0)} B_A &= \chi_{\pm A}, \quad Q_{\pm}^{(0)} \chi_{\pm A} = \pm [\phi_{\pm}, B_A], \\ Q_{\mp}^{(0)} \chi_{\pm A} &= -\frac{1}{2} [B_A, C] \mp H_A, \\ Q_{\pm}^{(0)} H_A &= [\phi_{\pm}, \chi_{\mp A}] \pm \frac{1}{2} [B_A, \eta_{\pm}] \mp \frac{1}{2} [C, \chi_{\pm A}], \end{aligned}$$

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<sup>7</sup> This is obtained from BTFT formulation of 4d  $\mathcal{N} = 4$  SYM in [18] by dimensional reduction. Here, we redefine  $H_A + \frac{1}{2} \epsilon_{ABC} [B_B, B_C]$ ,  $\phi$ ,  $\bar{\phi}$  in (4.13) in [18] as  $H_A$ ,  $\phi_+$ ,  $\phi_-$ , respectively.

$$\begin{aligned}
Q_{\pm}^{(0)} C &= \eta_{\pm}, \quad Q_{\pm}^{(0)} \eta_{\pm} = \pm [\phi_{\pm}, C], \\
Q_{\mp}^{(0)} \eta_{\pm} &= \mp [\phi_{\pm}, \phi_{\mp}], \\
Q_{\pm}^{(0)} \phi_{\pm} &= 0, \quad Q_{\mp}^{(0)} \phi_{\pm} = \mp \eta_{\pm}.
\end{aligned} \tag{14}$$

One can see the nilpotency  $(Q_{+}^{(0)})^2 = (Q_{-}^{(0)})^2 = \{Q_{+}^{(0)}, Q_{-}^{(0)}\} = 0$  up to gauge transformations.

We introduce a mass parameter  $\mu$  to deform these charges as [17]<sup>8</sup>

$$Q_{\pm} = Q_{\pm}^{(0)} + \Delta Q_{\pm}, \tag{15}$$

where non-vanishing  $\Delta Q_{\pm}$  transformations are

$$\begin{aligned}
\Delta Q_{\pm} \tilde{H}_i &= \frac{\mu}{3} \psi_{\pm i}, \quad \Delta Q_{\pm} H_A = \frac{\mu}{3} \chi_{\pm A}, \\
\Delta Q_{\pm} \eta_{\pm} &= \frac{2\mu}{3} \phi_{\pm}, \quad \Delta Q_{\mp} \eta_{\pm} = \pm \frac{\mu}{3} C.
\end{aligned} \tag{16}$$

Then  $Q_{\pm}$  satisfy the anti-commutation relations,

$$\begin{aligned}
Q_{+}^2 &= \frac{\mu}{3} J_{++}, \quad Q_{-}^2 = -\frac{\mu}{3} J_{--}, \\
\{Q_{+}, Q_{-}\} &= -\frac{\mu}{3} J_0,
\end{aligned} \tag{17}$$

up to gauge transformations, where  $J_0$ ,  $J_{++}$  and  $J_{--}$  are generators of  $SU(2)_R$  symmetry [18]. The eigenvalues of  $J_0$  are  $\pm 1$  for the fermions with index  $\pm$ ,  $\pm 2$  for  $\phi_{\pm}$ , and zero for the other bosonic fields. Note that  $\phi_{\pm}$  and  $C$  form an  $SU(2)_R$  triplet and each pair of  $(\psi_{+i}, \psi_{-i})$ ,  $(\chi_{+A}, \chi_{-A})$ ,  $(\eta_{+}, -\eta_{-})$  and  $(Q_{+}, Q_{-})$  forms a doublet. In particular,  $[J_{\pm\pm}, Q_{\pm}] = 0$ ,  $[J_{\pm\pm}, Q_{\mp}] = Q_{\pm}$ .

Using the modified supercharges, we can define  $Q_{\pm}$ -closed action as

$$S = \left( Q_{+} Q_{-} - \frac{\mu}{3} \right) \mathcal{F}, \tag{18}$$

where

$$\begin{aligned}
\mathcal{F} &= \mathcal{F}^{(0)} + \Delta \mathcal{F}, \\
\Delta \mathcal{F} &= \frac{1}{2g_{0d}^2} \text{Tr} \left( \frac{1}{2} \sum_{A=1}^3 a_A B_A^2 + \frac{1}{2} \sum_{i=1}^4 c_i X_i^2 \right).
\end{aligned} \tag{19}$$

That the action (18) is  $Q_{\pm}$ -closed can easily be seen by using (17) and the  $SU(2)_R$  invariance of  $\mathcal{F}$ . Here we take  $a_A = -\frac{2\mu}{3}$  and  $c_i = 0$  for convenience. After integrating out auxiliary fields, the action reads

$$S = S_0 + \Delta S, \tag{20}$$

where

$$\Delta S = \frac{1}{2g_{0d}^2} \text{Tr} \left\{ -\frac{\mu}{2} C[\phi_{+}, \phi_{-}] + \frac{\mu^2}{9} \left( \frac{C^2}{4} + \phi_{+} \phi_{-} \right) + \frac{2\mu}{3} \psi_{+i} \psi_{-i} - \frac{\mu}{6} \eta_{+} \eta_{-} - \frac{4\mu}{3} B_1[B_2, B_3] \right\}. \tag{21}$$

From this expression one can see some similarity to the plane wave matrix model [11]. Note that the term  $\text{Tr} B_1[B_2, B_3]$  is purely imaginary.

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<sup>8</sup> This is obtained by dimensionally reducing the algebra in two dimensions [17]. Note that, in contrast to the plane wave matrix model [11], supersymmetry transformation parameter does not depend on the coordinate and hence the dimensional reduction works.

## 5 Deconstructing 2d theory

### 5.1 Deconstructing 2d plane-wave SYM

In this section we construct a two-dimensional lattice through the deconstruction.

As already mentioned, the advantage of the deconstruction method is *if the orbifolding commutes with SUSY transformation generated by  $Q_\pm$ , it is promoted to the supersymmetry of the lattice*. To find such an orbifolding condition, we introduce complex fields

$$\begin{aligned} x &= X_1 + iX_2, & y &= X_3 + iX_4, \\ \xi_{\pm x} &= \psi_{\pm 1} + i\psi_{\pm 2}, & \xi_{\pm y} &= \psi_{\pm 3} + i\psi_{\pm 4}, \\ \tilde{h}_x &= \tilde{H}_1 + i\tilde{H}_2, & \tilde{h}_y &= \tilde{H}_3 + i\tilde{H}_4 \end{aligned} \quad (22)$$

and

$$b = B_1 + iB_2, \quad \rho_\pm = \chi_{\pm 1} + i\chi_{\pm 2}, \quad h = H_1 + iH_2. \quad (23)$$

Then we assign

$$\vec{r}_x = (1, 0) \quad (x, \xi_{\pm x} \text{ and } \tilde{h}_x), \quad (24)$$

$$\vec{r}_y = (0, 1) \quad (y, \xi_{\pm y} \text{ and } \tilde{h}_y), \quad (25)$$

$$\vec{r}_b = (-1, 1) \quad (b, \rho_\pm \text{ and } h). \quad (26)$$

For other fields,  $\vec{r}$  is taken to be zero. This charge assignment is similar to the one used in 2d  $\mathcal{N} = (8, 8)$  Kaplan-Ünsal model [24]. Compatibility with  $Q_\pm$  can easily be seen; fields with the same charge mixes linearly, up to multiplication of neutral fields. For example,  $x, \xi_{\pm x}$  and  $\tilde{h}_x$  transform as

$$\begin{aligned} Q_\pm^{(0)} x &= \xi_{\pm x}, \quad Q_\pm^{(0)} \xi_{\pm x} = \mp [x, \phi_\pm], \\ Q_\mp^{(0)} \xi_{\pm x} &= -\frac{1}{2} [x, C] \mp \tilde{h}_x, \\ Q_\pm^{(0)} \tilde{h}_x &= [\phi_\pm, \xi_{\mp x}] \mp \frac{1}{2} [C, \xi_{\pm x}] \pm \frac{1}{2} [x, \eta_\pm]. \end{aligned} \quad (27)$$

Neutrality of the action follows from the neutrality of  $\mathcal{F}$ . It can be seen by rewriting  $\mathcal{F}$  by complex notation; for example,  $\mathcal{F}^{(0)}$  becomes

$$\begin{aligned} \mathcal{F}^{(0)} &= \frac{1}{2g_{0d}^2} \text{Tr} \left\{ -\bar{b}[\bar{x}, y] - b[x, \bar{y}] + iB_3(-[x, \bar{x}] + [y, \bar{y}]) - iB_3[b, \bar{b}] \right. \\ &\quad \left. - \frac{\bar{\xi}_{+x}\xi_{-x} + \xi_{+x}\bar{\xi}_{-x}}{2} - \frac{\bar{\xi}_{+y}\xi_{-y} + \xi_{+y}\bar{\xi}_{-y}}{2} - \frac{\bar{\rho}_{+}\rho_{-} + \rho_{+}\bar{\rho}_{-}}{2} - \chi_{+3}\chi_{-3} - \frac{1}{4}\eta_{+}\eta_{-} \right\}, \end{aligned} \quad (28)$$

where  $\bar{\xi}_{\pm x} \equiv \psi_{\pm 1} - i\psi_{\pm 2}$ ,  $\bar{\xi}_{\pm y} \equiv \psi_{\pm 3} - i\psi_{\pm 4}$  and  $\bar{\rho}_\pm \equiv \chi_{\pm 1} - i\chi_{\pm 2}$ . Note that fields with bars have charges of the opposite sign.

By the deconstruction, fields with charge  $(1, 0)$  and  $(0, 1)$  become link variables on  $x$ - and  $y$ -directions, respectively, those with  $(-1, 1)$  become variables on  $(-\hat{x}, \hat{y})$  “diagonal” link, and those with  $(0, 0)$  become site variables.

In the following we will show only bosonic part. (Fermionic part is shown in appendix.) Mother theory is

$$\begin{aligned}
S_0^{bos} = & \frac{1}{2g_{0d}^2} Tr \left\{ \frac{1}{4} |[x, \bar{x}] + [y, \bar{y}]|^2 + |[x, y]|^2 + \frac{1}{4} |[\phi_+, \phi_-]|^2 + \frac{1}{4} |[\phi_+, C]|^2 + \frac{1}{4} |[b, \bar{b}]|^2 + |[b, B_3]|^2 \right. \\
& + \frac{1}{2} |[x, \phi_+]|^2 + \frac{1}{2} |[\bar{x}, \phi_+]|^2 + \frac{1}{4} |[x, C]|^2 + \frac{1}{2} |[y, \phi_+]|^2 + \frac{1}{2} |[\bar{y}, \phi_+]|^2 + \frac{1}{4} |[y, C]|^2 \\
& + \frac{1}{2} |[x, b]|^2 + \frac{1}{2} |[\bar{x}, b]|^2 + |[x, B_3]|^2 + \frac{1}{2} |[y, b]|^2 + \frac{1}{2} |[\bar{y}, b]|^2 + |[y, B_3]|^2 \\
& \left. + \frac{1}{4} |[b, C]|^2 + \frac{1}{2} |[b, \phi_+]|^2 + \frac{1}{2} |[\bar{b}, \phi_+]|^2 + \frac{1}{4} |[B_3, C]|^2 + |[B_3, \phi_+]|^2 \right\}, \quad (29)
\end{aligned}$$

$$\Delta S^{bos} = \frac{1}{2g_{0d}^2} Tr \left\{ -\frac{\mu}{2} C[\phi_+, \phi_-] + \frac{\mu^2}{9} \left( \frac{C^2}{4} + \phi_+ \phi_- - \frac{2i\mu}{3} B_3[b, \bar{b}] \right) \right\}. \quad (30)$$

After projection it reduces to

$$\begin{aligned}
S_0^{bos,lat} = & \frac{1}{2g_{0d}^2} \sum_{\vec{n}} Tr \left\{ \frac{1}{4} |x_{\vec{n}} \bar{x}_{\vec{n}} - \bar{x}_{\vec{n}-\hat{x}} x_{\vec{n}-\hat{x}} + y_{\vec{n}} \bar{y}_{\vec{n}} - \bar{y}_{\vec{n}-\hat{y}} y_{\vec{n}-\hat{y}}|^2 + |x_{\vec{n}} y_{\vec{n}+\hat{x}} - y_{\vec{n}} x_{\vec{n}+\hat{y}}|^2 \right. \\
& + \frac{1}{4} |[\phi_+, \vec{n}, \phi_-, \vec{n}]|^2 + \frac{1}{4} |[\phi_+, \vec{n}, C_{\vec{n}}]|^2 + \frac{1}{4} |b_{\vec{n}} \bar{b}_{\vec{n}} - \bar{b}_{\vec{n}+\hat{x}-\hat{y}} b_{\vec{n}+\hat{x}-\hat{y}}|^2 + |b_{\vec{n}} B_{3, \vec{n}-\hat{x}+\hat{y}} - B_{3, \vec{n}} b_{\vec{n}}|^2 \\
& + \frac{1}{2} |x_{\vec{n}} \phi_{+, \vec{n}+\hat{x}} - \phi_{+, \vec{n}} x_{\vec{n}}|^2 + \frac{1}{2} |\bar{x}_{\vec{n}-\hat{x}} \phi_{+, \vec{n}-\hat{x}} - \phi_{+, \vec{n}} \bar{x}_{\vec{n}-\hat{x}}|^2 + \frac{1}{4} |x_{\vec{n}} C_{\vec{n}+\hat{x}} - C_{\vec{n}} x_{\vec{n}}|^2 \\
& + \frac{1}{2} |y_{\vec{n}} \phi_{+, \vec{n}+\hat{y}} - \phi_{+, \vec{n}} y_{\vec{n}}|^2 + \frac{1}{2} |\bar{y}_{\vec{n}-\hat{y}} \phi_{+, \vec{n}-\hat{y}} - \phi_{+, \vec{n}} \bar{y}_{\vec{n}-\hat{y}}|^2 + \frac{1}{4} |y_{\vec{n}} C_{\vec{n}+\hat{y}} - C_{\vec{n}} y_{\vec{n}}|^2 \\
& + \frac{1}{2} |x_{\vec{n}} b_{\vec{n}+\hat{x}} - b_{\vec{n}} x_{\vec{n}-\hat{x}+\hat{y}}|^2 + \frac{1}{2} |\bar{x}_{\vec{n}} b_{\vec{n}} - b_{\vec{n}+\hat{x}} \bar{x}_{\vec{n}-\hat{x}+\hat{y}}|^2 + |x_{\vec{n}} B_{3, \vec{n}+\hat{x}} - B_{3, \vec{n}} x_{\vec{n}}|^2 \\
& + \frac{1}{2} |y_{\vec{n}} b_{\vec{n}+\hat{y}} - b_{\vec{n}} y_{\vec{n}-\hat{x}+\hat{y}}|^2 + \frac{1}{2} |\bar{y}_{\vec{n}-\hat{y}} b_{\vec{n}-\hat{y}} - b_{\vec{n}} \bar{y}_{\vec{n}-\hat{x}}|^2 + |y_{\vec{n}} B_{3, \vec{n}+\hat{y}} - B_{3, \vec{n}} y_{\vec{n}}|^2 \\
& + \frac{1}{4} |b_{\vec{n}} C_{\vec{n}-\hat{x}+\hat{y}} - C_{\vec{n}} b_{\vec{n}}|^2 + \frac{1}{2} |b_{\vec{n}} \phi_{+, \vec{n}-\hat{x}+\hat{y}} - \phi_{+, \vec{n}} b_{\vec{n}}|^2 + \frac{1}{2} |\bar{b}_{\vec{n}} \phi_{+, \vec{n}} - \phi_{+, \vec{n}-\hat{x}+\hat{y}} \bar{b}_{\vec{n}}|^2 \\
& \left. + \frac{1}{4} |[B_{3, \vec{n}}, C_{\vec{n}}]|^2 + |[B_{3, \vec{n}}, \phi_{+, \vec{n}}]|^2 \right\} \quad (31)
\end{aligned}$$

and

$$\Delta S^{bos} = \frac{1}{2g_{0d}^2} \sum_{\vec{n}} Tr \left\{ -\frac{\mu}{2} C_{\vec{n}}[\phi_+, \vec{n}, \phi_-, \vec{n}] + \frac{\mu^2}{9} \left( \frac{C_{\vec{n}}^2}{4} + \phi_{+, \vec{n}} \phi_{-, \vec{n}} - \frac{2i\mu}{3} B_{3, \vec{n}} (b_{\vec{n}} \bar{b}_{\vec{n}} - \bar{b}_{\vec{n}+\hat{x}-\hat{y}} b_{\vec{n}+\hat{x}-\hat{y}}) \right) \right\}. \quad (32)$$

SUSY transformation on lattice is shown in appendix.

If we expand this model around  $x_{\vec{n}} \bar{x}_{\vec{n}} = y_{\vec{n}} \bar{y}_{\vec{n}} = \frac{1}{a^2} \cdot \mathbf{1}$ , 2d SYM is obtained. To stabilize the background, we add soft SUSY breaking terms<sup>9</sup>. Here we add two kinds of mass terms,

$$\frac{a^2}{8g_{0d}^2} \sum_{\vec{n}} \left\{ \nu_1^2 Tr \left( \left| x_{\vec{n}} \bar{x}_{\vec{n}} - \frac{1}{a^2} \right|^2 + \left| y_{\vec{n}} \bar{y}_{\vec{n}} - \frac{1}{a^2} \right|^2 \right) + \nu_2^2 \left( \left| \frac{Tr(x_{\vec{n}} \bar{x}_{\vec{n}})}{N} - \frac{1}{a^2} \right|^2 + \left| \frac{Tr(y_{\vec{n}} \bar{y}_{\vec{n}})}{N} - \frac{1}{a^2} \right|^2 \right) \right\}. \quad (33)$$

<sup>9</sup> In principle we can obtain such term keeping two supersymmetries, by adding appropriate terms to  $\mathcal{F}$ , though the action becomes ugly. For simulation, it might be necessary to add mass for  $b, \bar{b}$  and  $B_3$  as well. It can be done keeping two exact supersymmetries by changing the values of  $a_A$  in (19); adding soft SUSY breaking mass is also fine. In any case the following argument for the absence of fine tuning is not modified.



Two-dimensional coupling constant  $g_{2d}$  is given as before,

$$g_{2d}^2 = a^2 g_{0d}^2. \quad (34)$$

A parameter  $\nu_1$  gives mass of both  $SU(N)$  and  $U(1)$  parts, while  $\nu_2$  is solely  $U(1)$  mass<sup>10</sup>. As argued in [9] and [38],  $SU(N)$  flat direction is lifted by quantum effects and nonabelian phase (i.e. a phase in which all scalar eigenvalues localizes to a point) becomes stable at large- $N$ . Therefore, we can send  $\nu_1$  to zero when we consider the large- $N$  limit. On the other hand, to stabilize  $U(1)$  flat direction, we must take  $\nu_2$  to be  $O(1)$ . However it does not affect the supersymmetry in  $SU(N)$  sector, because in the continuum limit  $U(1)$  part is just a decoupled free sector.

Note that we can introduce  $U(1)$  mass for other scalars as well, although those  $U(1)$  modes are harmless.

## 5.2 Continuum limit and absence of fine tuning

At tree level, continuum action is<sup>11</sup> (here we show only bosonic part; Also we omitted  $U(1)$  sector.)

$$\begin{aligned} S^{bos, cont} = & \frac{1}{g_{2d}^2} \int d^2x \text{Tr} \left\{ \frac{1}{2} F_{12}^2 + \frac{1}{2} (D_\mu X_I)^2 - \frac{1}{4} [X_I, X_J]^2 + \frac{\mu^2}{18} \sum_{a=1}^3 X_a^2 + 2i\mu X_1 [X_2, X_3] \right. \\ & \left. - \frac{4\mu}{3} X_6 [X_7, X_8] + \frac{\nu_1^2}{2} \sum_{i=4}^5 X_i^2 \right\}. \end{aligned} \quad (35)$$

When  $\nu_1 = 0$ , deformation by  $\mu$  breaks 14 out of 16 SUSY softly. With nonzero  $\nu_1$  remaining two is softly broken as well.

In the following we show that, at perturbative level, radiative corrections from UV region do not change this continuum action, except for the soft terms due to nonzero value of  $\nu_1$ . Because our lattice has the same symmetries as the model in [17], the argument goes completely in parallel. Firstly let us consider the case that  $\nu_1 = 0$ . Let us consider operators of the form  $\mathcal{O}_p = \varphi^\alpha \partial^\beta \phi^{2\gamma}$ , whose mass dimension is  $p = \alpha + \beta + 3\gamma$ . Here  $\varphi$  and  $\phi$  stand for boson and fermion, respectively. Because the coupling constant  $g_{2d}$  is dimensionful, the correction is of the form

$$(c_1 a^{p-2} + g_{2d}^2 c_2 a^p + \dots) \int d^2x \mathcal{O}_p(x). \quad (36)$$

Here  $a^{p-4}/g_{2d}^2$  is omitted because it is a tree-level contribution. Therefore, nonzero contribution may remain in the continuum limit  $a \rightarrow 0$  only when  $p = 1, 2$ , and if that happens  $\mathcal{O}$  must be  $\varphi$  or  $\varphi^2$ . But all such terms are forbidden by exact  $Q_\pm$  SUSY and  $SU(2)$  R-symmetry. Even if  $\nu_1$  is not zero, it is just a soft mass, which does not alter UV divergence. Therefore in an appropriate large-volume and  $\nu_1 \rightarrow 0$  limit SUSY breaking effect disappears [23, 24]. When we consider an uplift to 4d theory in the following,  $N$  is sent to infinity, and then scalar flat direction is lifted by the quantum effect. Therefore we can take  $\nu_1$  to be zero.

<sup>10</sup> We would like to thank O. Aharony for his suggestion of  $U(1)$  mass term.

<sup>11</sup> Here  $X_i$  stand for  $X_1 = (\phi_+ + \phi_-)/2$ ,  $X_2 = (\phi_+ - \phi_-)/2i$ ,  $X_3 = C/2$ ,  $X_{4,5} = s_{1,2}$ ,  $X_{6,7,8} = B_{1,2,3}$ .

### 5.3 Remarks

The argument provided in this section applies to all order in perturbation theory. Whether the absence of the sign problem persists to nonperturbative level should be checked by numerics. In four-SUSY cousin of this model the absence of the fine tuning at nonperturbative level has been confirmed in [38, 7] for two independent formulations by Sugino and by Cohen-Kaplan-Katz-Unsal (the latter corresponds to the one adopted in this paper), by treating fermions dynamically. It is natural to expect that the absence of fine tuning persists to nonperturbative level also in the present case with maximal supersymmetry.

However, with maximal supersymmetry, there is a possible difficulty for the Monte-Carlo simulation – Pfaffian of the Dirac operator is complex in general and hence usual Markov chain Monte-Carlo method cannot be used. At finite temperature, the phase is almost absent even at rather low temperature which is physically interesting, and phase quench approximation, in which the Pfaffian is replaced by its absolute value, gives a good approximation [9, 10, 44]. (Strictly speaking, the vacuum studied in these works are different from the fuzzy sphere. Also, in order to obtain 4d theory one has to take a certain scaling limit. Therefore, whether the phase is small in the present case must be checked independently.) It enable us to test the validity of the model at nonperturbative level. Surprisingly, in one dimension, even at very low temperature and/or with SUSY-preserving boundary condition, where the phase fluctuates, the phase quench approximation provide a correct results predicted by the gauge/gravity duality [9]<sup>12</sup>. It is interesting to check whether the same happens in the present case.

## 6 Uplift to 4d

For the reason explained above, we assume  $\nu_1 = 0$ . The continuum action (35) has constant BPS fuzzy sphere solution<sup>13</sup>

$$X_a(x) = \frac{\mu}{3}L_a \quad (a = 1, 2, 3), \quad X_i(x) = 0 \quad (i = 4, \dots, 8), \quad (37)$$

where  $L_a$  are  $M \times M$  matrices satisfying  $SU(2)$  commutation relation

$$[L_a, L_b] = i\epsilon_{abc}L_c. \quad (38)$$

By taking  $k$ -coincident fuzzy sphere solution,  $L_a = L_a^{(M/k)} \otimes \mathbf{1}_k$ , where  $L_a^{(M/k)}$  is the  $(M/k) \times (M/k)$  irreducible representation, we obtain 4d  $U(k)$  theory on fuzzy sphere. Essentially, adjoint action of  $L_a$  is identified with the derivative and  $[X_a, \cdot]$  is regarded as the gauge covariant derivative [39]. The noncommutativity is given by  $\theta \sim k/(\mu^2 M)$  and UV/IR momentum cutoffs along spherical directions are  $\mu M/k$  and  $\mu$ , respectively. 4d coupling is given by  $g_{4d}^2 = 4\pi\theta g_{2d}^2$ . In order to get continuum 4d theory, we take large- $M$  and small  $\mu$  limit while fixing  $k$  and  $g_{4d}^2$ . In that limit, maximal supersymmetry is restored because soft SUSY breaking parameter  $\mu$  goes to zero. One can take a limit with any value of noncommutativity  $\theta$ , and  $\theta \rightarrow 0$  limit is expected to be smooth [30, 31]. That the limit should be smooth is natural physically, because a possible obstacle is a new IR divergence arising due to the UV/IR mixing reflecting the UV divergence, which should

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<sup>12</sup>Numerically it can be justified by looking at the correlation between the phase and values of observables.

<sup>13</sup> This background preserves exact supersymmetries at discretized level.

be absent in UV finite theories. However there is no rigorous proof in mathematical sense. Our formulation itself can serve as a nonperturbative framework to check the smoothness.

In the above we assumed the radius of the fuzzy sphere does not deviate from classical value. Whether it is the case or not should be tested by numerics. If the radius is renormalized, we should take into account it by replacing the parameters in the mapping rule with renormalized ones.

## 7 Conclusion and discussions

In this paper we proposed a nonperturbative regularization of 4d  $\mathcal{N} = 4$  SYM which does not require parameter fine tuning at least at perturbative level. It is much simpler than similar model [17] and can easily be put on computer. Therefore absence of the fine tuning at nonperturbative level can be tested more easily. Except for the lack of a mathematical proof of the smoothness of the commutative limit of the noncommutative space, which can be tested numerically by using this model itself, this method provides the first formulation of four-dimensional extended SYM free from the fine tuning at perturbative level. Note that in other models absence of fine tuning is not shown even at perturbative level. We expect lattice Monte-Carlo simulation will be performed in near future and new insights into  $AdS_5/CFT_4$  correspondence will be obtained.

Soft mass term for  $U(1)$  scalar introduced in (33) is very simple but can play an important role. Consider the original, non-deformed model. As already mentioned,  $SU(N)$  flat direction is lifted at large- $N$ , and hence  $U(N)$  mass term  $\nu_1$  can be set to zero. Therefore, even at finite volume, maximal supersymmetry is fully restored. Such a finite volume theory has a gravity dual description [41] of the black hole/black string transition [40]. This theory is expected to have a rich phase structure as a function of volume and temperature [41, 42], and details of the geometry of the transition can be studied from Monte-Carlo data [43]. Hence Monte-Carlo simulation would provide valuable insights into the stringy correction to the transition<sup>14</sup>.

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## A Fermionic part of the lattice action

Fermionic part of the two-dimensional lattice action is written as

$$S^{fer} = S_0^{fer} + \Delta S^{fer}, \quad S_0^{fer} = \sum_{i=1}^{10} \frac{1}{2g_{0d}^2} \sum_{\vec{n}} \mathcal{L}_i, \quad (39)$$

where

$$\mathcal{L}_1 = -\bar{\rho}_{+, \vec{n}} \left( (\bar{\xi}_{-x, \vec{n}-\hat{x}} y_{\vec{n}-\hat{x}} - y_{\vec{n}} \bar{\xi}_{-x, \vec{n}-\hat{x}+\hat{y}}) + (\bar{x}_{\vec{n}-\hat{x}} \xi_{-y, \vec{n}-\hat{x}} - \xi_{-y, \vec{n}} \bar{x}_{\vec{n}-\hat{x}+\hat{y}}) \right)$$

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<sup>14</sup> For recent simulations in this context, see [38, 44].

$$+ \bar{\rho}_{-, \bar{n}} ((\bar{\xi}_{+x, \bar{n}-\hat{x}} y_{\bar{n}-\hat{x}} - y_{\bar{n}} \bar{\xi}_{+x, \bar{n}-\hat{x}+\hat{y}}) + (\bar{x}_{\bar{n}-\hat{x}} \xi_{+y, \bar{n}-\hat{x}} - \xi_{+y, \bar{n}} \bar{x}_{\bar{n}-\hat{x}+\hat{y}})) \\ - \bar{b}_{\bar{n}} ((\bar{\xi}_{+x, \bar{n}-\hat{x}} \xi_{-y, \bar{n}-\hat{x}} + \xi_{-y, \bar{n}} \bar{\xi}_{+x, \bar{n}-\hat{x}+\hat{y}}) - (\bar{\xi}_{-x, \bar{n}-\hat{x}} \xi_{+y, \bar{n}-\hat{x}} + \xi_{+y, \bar{n}} \bar{\xi}_{-x, \bar{n}-\hat{x}+\hat{y}})) , \quad (40)$$

$$\mathcal{L}_2 = -\rho_{+, \bar{n}+\hat{x}-\hat{y}} ((\xi_{-x, \bar{n}} \bar{y}_{\bar{n}+\hat{x}-\hat{y}} - \bar{y}_{\bar{n}-\hat{y}} \xi_{-x, \bar{n}-\hat{y}}) + (x_{\bar{n}} \bar{\xi}_{-y, \bar{n}+\hat{x}-\hat{y}} - \bar{\xi}_{-y, \bar{n}-\hat{y}} x_{\bar{n}-\hat{y}})) \\ + \rho_{-, \bar{n}+\hat{x}-\hat{y}} ((\xi_{+x, \bar{n}} \bar{y}_{\bar{n}+\hat{x}-\hat{y}} - \bar{y}_{\bar{n}-\hat{y}} \xi_{+x, \bar{n}-\hat{y}}) + (x_{\bar{n}} \bar{\xi}_{+y, \bar{n}+\hat{x}-\hat{y}} - \bar{\xi}_{+y, \bar{n}-\hat{y}} x_{\bar{n}-\hat{y}})) \\ - b_{\bar{n}+\hat{x}-\hat{y}} ((\xi_{+x, \bar{n}} \bar{\xi}_{-y, \bar{n}+\hat{x}-\hat{y}} + \bar{\xi}_{-y, \bar{n}-\hat{y}} \xi_{+x, \bar{n}-\hat{y}}) - (\xi_{-x, \bar{n}} \bar{\xi}_{+y, \bar{n}+\hat{x}-\hat{y}} + \bar{\xi}_{+y, \bar{n}-\hat{y}} \xi_{-x, \bar{n}-\hat{y}})) , \quad (41)$$

$$\mathcal{L}_3 = -i\chi_{+3, \bar{n}} ((\xi_{-x, \bar{n}} \bar{x}_{\bar{n}} - \bar{x}_{\bar{n}-\hat{x}} \xi_{-x, \bar{n}-\hat{x}}) + (x_{\bar{n}} \bar{\xi}_{-x, \bar{n}} - \bar{\xi}_{-x, \bar{n}-\hat{x}} x_{\bar{n}-\hat{x}})) \\ + i\chi_{-3, \bar{n}} ((\xi_{+x, \bar{n}} \bar{x}_{\bar{n}} - \bar{x}_{\bar{n}-\hat{x}} \xi_{+x, \bar{n}-\hat{x}}) + (x_{\bar{n}} \bar{\xi}_{+x, \bar{n}} - \bar{\xi}_{+x, \bar{n}-\hat{x}} x_{\bar{n}-\hat{x}})) \\ - iB_{3, \bar{n}} ((\xi_{+x, \bar{n}} \bar{\xi}_{-x, \bar{n}} + \bar{\xi}_{-x, \bar{n}-\hat{x}} \xi_{+x, \bar{n}-\hat{x}}) - (\xi_{-x, \bar{n}} \bar{\xi}_{+x, \bar{n}} + \bar{\xi}_{+x, \bar{n}-\hat{x}} \xi_{-x, \bar{n}-\hat{x}})) , \quad (42)$$

$$\mathcal{L}_4 = i\chi_{+3, \bar{n}} ((\xi_{-y, \bar{n}} \bar{y}_{\bar{n}} - \bar{y}_{\bar{n}-\hat{y}} \xi_{-y, \bar{n}-\hat{y}}) + (y_{\bar{n}} \bar{\xi}_{-y, \bar{n}} - \bar{\xi}_{-y, \bar{n}-\hat{y}} y_{\bar{n}-\hat{y}})) \\ - i\chi_{-3, \bar{n}} ((\xi_{+y, \bar{n}} \bar{y}_{\bar{n}} - \bar{y}_{\bar{n}-\hat{y}} \xi_{+y, \bar{n}-\hat{y}}) + (y_{\bar{n}} \bar{\xi}_{+y, \bar{n}} - \bar{\xi}_{+y, \bar{n}-\hat{y}} y_{\bar{n}-\hat{y}})) \\ + iB_{3, \bar{n}} ((\xi_{+y, \bar{n}} \bar{\xi}_{-y, \bar{n}} + \bar{\xi}_{-y, \bar{n}-\hat{y}} \xi_{+y, \bar{n}-\hat{y}}) - (\xi_{-y, \bar{n}} \bar{\xi}_{+y, \bar{n}} + \bar{\xi}_{+y, \bar{n}-\hat{y}} \xi_{-y, \bar{n}-\hat{y}})) , \quad (43)$$

$$\mathcal{L}_5 = -i\chi_{+3, \bar{n}} ((\rho_{-, \bar{n}} \bar{b}_{\bar{n}} - \bar{b}_{\bar{n}+\hat{x}-\hat{y}} \rho_{-, \bar{n}+\hat{x}-\hat{y}}) + (b_{\bar{n}} \bar{\rho}_{-, \bar{n}} - \bar{\rho}_{-, \bar{n}+\hat{x}-\hat{y}} b_{\bar{n}+\hat{x}-\hat{y}})) \\ + i\chi_{-3, \bar{n}} ((\rho_{+, \bar{n}} \bar{b}_{\bar{n}} - \bar{b}_{\bar{n}+\hat{x}-\hat{y}} \rho_{+, \bar{n}+\hat{x}-\hat{y}}) + (b_{\bar{n}} \bar{\rho}_{+, \bar{n}} - \bar{\rho}_{+, \bar{n}+\hat{x}-\hat{y}} b_{\bar{n}+\hat{x}-\hat{y}})) \\ - iB_{3, \bar{n}} ((\rho_{+, \bar{n}} \bar{\rho}_{-, \bar{n}} + \bar{\rho}_{-, \bar{n}+\hat{x}-\hat{y}} \rho_{+, \bar{n}+\hat{x}-\hat{y}}) - (\rho_{-, \bar{n}} \bar{\rho}_{+, \bar{n}} + \bar{\rho}_{+, \bar{n}+\hat{x}-\hat{y}} \rho_{-, \bar{n}+\hat{x}-\hat{y}})) , \quad (44)$$

$$\mathcal{L}_6 = -\sum_{\bar{n}} \frac{1}{2} \left\{ 2\bar{\xi}_{+x, \bar{n}} (\xi_{+x, \bar{n}} \phi_{-, \bar{n}+\hat{x}} - \phi_{-, \bar{n}} \xi_{+x, \bar{n}}) \right. \\ + \bar{\xi}_{+x, \bar{n}} (x_{\bar{n}} \eta_{-, \bar{n}+\hat{x}} - \eta_{-, \bar{n}} x_{\bar{n}}) + \xi_{+x, \bar{n}} (\bar{x}_{\bar{n}} \eta_{-, \bar{n}} - \eta_{-, \bar{n}+\hat{x}} \bar{x}_{\bar{n}}) \\ + ((C_{\bar{n}+\hat{x}} \bar{\xi}_{+x, \bar{n}} - \bar{\xi}_{+x, \bar{n}} C_{\bar{n}}) - (\bar{x}_{\bar{n}} \eta_{+, \bar{n}} - \eta_{+, \bar{n}+\hat{x}} \bar{x}_{\bar{n}}) - (\phi_{+, \bar{n}+\hat{x}} \bar{\xi}_{-x, \bar{n}} - \bar{\xi}_{-x, \bar{n}} \phi_{+, \bar{n}})) \xi_{-x, \bar{n}} \\ \left. + ((C_{\bar{n}} \xi_{+x, \bar{n}} - \xi_{+x, \bar{n}} C_{\bar{n}+\hat{x}}) - (x_{\bar{n}} \eta_{+, \bar{n}+\hat{x}} - \eta_{+, \bar{n}} x_{\bar{n}}) - (\phi_{+, \bar{n}} \xi_{-x, \bar{n}} - \xi_{-x, \bar{n}} \phi_{+, \bar{n}+\hat{x}})) \bar{\xi}_{-x, \bar{n}} \right\} , \quad (45)$$

$$\mathcal{L}_7 = -\sum_{\bar{n}} \frac{1}{2} \left\{ 2\bar{\xi}_{+y, \bar{n}} (\xi_{+y, \bar{n}} \phi_{-, \bar{n}+\hat{y}} - \phi_{-, \bar{n}} \xi_{+y, \bar{n}}) \right. \\ + \bar{\xi}_{+y, \bar{n}} (y_{\bar{n}} \eta_{-, \bar{n}+\hat{y}} - \eta_{-, \bar{n}} y_{\bar{n}}) + \xi_{+y, \bar{n}} (\bar{y}_{\bar{n}} \eta_{-, \bar{n}} - \eta_{-, \bar{n}+\hat{y}} \bar{y}_{\bar{n}}) \\ + ((C_{\bar{n}+\hat{y}} \bar{\xi}_{+y, \bar{n}} - \bar{\xi}_{+y, \bar{n}} C_{\bar{n}}) - (\bar{y}_{\bar{n}} \eta_{+, \bar{n}} - \eta_{+, \bar{n}+\hat{y}} \bar{y}_{\bar{n}}) - (\phi_{+, \bar{n}+\hat{y}} \bar{\xi}_{-y, \bar{n}} - \bar{\xi}_{-y, \bar{n}} \phi_{+, \bar{n}})) \xi_{-y, \bar{n}} \\ \left. + ((C_{\bar{n}} \xi_{+y, \bar{n}} - \xi_{+y, \bar{n}} C_{\bar{n}+\hat{y}}) - (y_{\bar{n}} \eta_{+, \bar{n}+\hat{y}} - \eta_{+, \bar{n}} y_{\bar{n}}) - (\phi_{+, \bar{n}} \xi_{-y, \bar{n}} - \xi_{-y, \bar{n}} \phi_{+, \bar{n}+\hat{y}})) \bar{\xi}_{-y, \bar{n}} \right\} ,$$

(46)

$$\begin{aligned}
\mathcal{L}_8 = & - \sum_{\vec{n}} \frac{1}{2} \left\{ 2\bar{\rho}_{+, \vec{n}}(\rho_{+, \vec{n}}\phi_{-, \vec{n}-\hat{x}+\hat{y}} - \phi_{-, \vec{n}}\rho_{-, \vec{n}}) \right. \\
& + \bar{\rho}_{+, \vec{n}}(b_{\vec{n}}\eta_{-, \vec{n}-\hat{x}+\hat{y}} - \eta_{-, \vec{n}}b_{\vec{n}}) + \rho_{+, \vec{n}}(\bar{b}_{\vec{n}}\eta_{-, \vec{n}} - \eta_{-, \vec{n}-\hat{x}+\hat{y}}\bar{b}_{\vec{n}}) \\
& + ((C_{\vec{n}-\hat{x}+\hat{y}}\bar{\rho}_{+, \vec{n}} - \bar{\rho}_{+, \vec{n}}C_{\vec{n}}) - (\bar{b}_{\vec{n}}\eta_{+, \vec{n}} - \eta_{+, \vec{n}-\hat{x}+\hat{y}}\bar{b}_{\vec{n}}) - (\phi_{+, \vec{n}-\hat{x}+\hat{y}}\bar{\rho}_{-, \vec{n}} - \bar{\rho}_{-, \vec{n}}\phi_{+, \vec{n}})) \rho_{-, \vec{n}} \\
& \left. + ((C_{\vec{n}}\rho_{+, \vec{n}} - \rho_{+, \vec{n}}C_{\vec{n}-\hat{x}+\hat{y}}) - (b_{\vec{n}}\eta_{+, \vec{n}-\hat{x}+\hat{y}} - \eta_{+, \vec{n}}b_{\vec{n}}) - (\phi_{+, \vec{n}}\rho_{-, \vec{n}} - \rho_{-, \vec{n}}\phi_{+, \vec{n}-\hat{x}+\hat{y}})) \bar{\rho}_{-, \vec{n}} \right\}, \tag{47}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_9 = & \sum_{\vec{n}} \left\{ \chi_{+, \vec{n}}[\eta_{-, \vec{n}}, B_{3, \vec{n}}] + \chi_{-, \vec{n}}[\eta_{+, \vec{n}}, B_{3, \vec{n}}] \right. \\
& \left. + \chi_{+, \vec{n}}[\phi_{-, \vec{n}}, \chi_{+, \vec{n}}] - \chi_{-, \vec{n}}[\phi_{+, \vec{n}}, \chi_{-, \vec{n}}] + [\chi_{+, \vec{n}}, C_{\vec{n}}]\chi_{-, \vec{n}} \right\}, \tag{48}
\end{aligned}$$

$$\mathcal{L}_{10} = \sum_{\vec{n}} \frac{1}{4} \left\{ \eta_{+, \vec{n}}[\eta_{-, \vec{n}}, C_{\vec{n}}] + \eta_{+, \vec{n}}[\phi_{-, \vec{n}}, \eta_{+, \vec{n}}] - \eta_{-, \vec{n}}[\phi_{+, \vec{n}}, \eta_{-, \vec{n}}] \right\} \tag{49}$$

and

$$\Delta S^{fer} = \frac{\mu}{2g_{0d}^2} \sum_{\vec{n}} Tr \left\{ \frac{1}{3} (\bar{\xi}_{+, \vec{n}}\xi_{-, \vec{n}} + \xi_{+, \vec{n}}\bar{\xi}_{-, \vec{n}} + \bar{\xi}_{+, \vec{n}}\xi_{-, \vec{n}} + \xi_{+, \vec{n}}\bar{\xi}_{-, \vec{n}}) - \frac{1}{6}\eta_{+, \vec{n}}\eta_{-, \vec{n}} \right\}. \tag{50}$$

## B SUSY transformation on lattice

SUSY transformation on two-dimensional lattice is given by

$$\begin{aligned}
Q_{\pm}^{(0)} x_{\vec{n}} &= \xi_{\pm x, \vec{n}}, \quad Q_{\pm}^{(0)} \xi_{\pm x, \vec{n}} = \mp (x_{\vec{n}}\phi_{\pm, \vec{n}+\hat{x}} - \phi_{\pm, \vec{n}}x_{\vec{n}}), \\
Q_{\mp}^{(0)} \xi_{\pm x, \vec{n}} &= -\frac{1}{2}(x_{\vec{n}}C_{\vec{n}+\hat{x}} - C_{\vec{n}}x_{\vec{n}}) \mp \tilde{h}_{x, \vec{n}}, \\
Q_{\pm}^{(0)} y_{\vec{n}} &= \xi_{\pm y, \vec{n}}, \quad Q_{\pm}^{(0)} \xi_{\pm y, \vec{n}} = \mp (y_{\vec{n}}\phi_{\pm, \vec{n}+\hat{y}} - \phi_{\pm, \vec{n}}y_{\vec{n}}), \\
Q_{\mp}^{(0)} \xi_{\pm y, \vec{n}} &= -\frac{1}{2}(y_{\vec{n}}C_{\vec{n}+\hat{y}} - C_{\vec{n}}y_{\vec{n}}) \mp \tilde{h}_{y, \vec{n}}, \\
Q_{\pm}^{(0)} \tilde{h}_{x, \vec{n}} &= (\phi_{\pm, \vec{n}}\xi_{\mp x, \vec{n}} - \xi_{\mp x, \vec{n}}\phi_{\pm, \vec{n}+\hat{x}}) \mp \frac{1}{2}(C_{\vec{n}}\xi_{\pm x, \vec{n}} - \xi_{\pm x, \vec{n}}C_{\vec{n}+\hat{x}}) \pm \frac{1}{2}(x_{\vec{n}}\eta_{\pm, \vec{n}+\hat{x}} - \eta_{\pm, \vec{n}}x_{\vec{n}}), \\
Q_{\pm}^{(0)} \tilde{h}_{y, \vec{n}} &= (\phi_{\pm, \vec{n}}\xi_{\mp y, \vec{n}} - \xi_{\mp y, \vec{n}}\phi_{\pm, \vec{n}+\hat{y}}) \mp \frac{1}{2}(C_{\vec{n}}\xi_{\pm y, \vec{n}} - \xi_{\pm y, \vec{n}}C_{\vec{n}+\hat{y}}) \pm \frac{1}{2}(y_{\vec{n}}\eta_{\pm, \vec{n}+\hat{y}} - \eta_{\pm, \vec{n}}y_{\vec{n}}), \\
Q_{\pm}^{(0)} b_{\vec{n}} &= \rho_{\pm, \vec{n}}, \quad Q_{\pm}^{(0)} \rho_{\pm, \vec{n}} = \pm (\phi_{\pm, \vec{n}}b_{\vec{n}} - b_{\vec{n}}\phi_{\pm, \vec{n}-\hat{x}+\hat{y}}), \\
Q_{\pm}^{(0)} B_{3, \vec{n}} &= \chi_{\pm 3, \vec{n}}, \quad Q_{\pm}^{(0)} \chi_{\pm 3, \vec{n}} = \pm [\phi_{\pm, \vec{n}}, B_{3, \vec{n}}], \\
Q_{\mp}^{(0)} \rho_{\pm, \vec{n}} &= -\frac{1}{2}(b_{\vec{n}}C_{\vec{n}-\hat{x}+\hat{y}} - C_{\vec{n}}b_{\vec{n}}) \mp h_{\vec{n}}
\end{aligned}$$

$$\begin{aligned}
Q_{\mp}^{(0)} \chi_{\pm 3, \vec{n}} &= -\frac{1}{2} [B_{3, \vec{n}}, C_{\vec{n}}] \mp H_{3, \vec{n}}, \\
Q_{\pm}^{(0)} h_{\vec{n}} &= (\phi_{\pm, \vec{n}} \rho_{\mp, \vec{n}} - \rho_{\mp, \vec{n}} \phi_{\pm, \vec{n}}) \pm \frac{1}{2} (b_{\vec{n}} \eta_{\pm, \vec{n}} - \eta_{\pm, \vec{n}} b_{\vec{n}}) \mp \frac{1}{2} (C_{\vec{n}} \rho_{\pm, \vec{n}} - \rho_{\pm, \vec{n}} C_{\vec{n}}), \\
Q_{\pm}^{(0)} H_{3, \vec{n}} &= [\phi_{\pm, \vec{n}}, \chi_{\mp 3, \vec{n}}] \pm \frac{1}{2} [B_{3, \vec{n}}, \eta_{\pm, \vec{n}}] \mp \frac{1}{2} [C_{\vec{n}}, \chi_{\pm 3, \vec{n}}], \\
Q_{\pm}^{(0)} C_{\vec{n}} &= \eta_{\pm, \vec{n}}, \quad Q_{\pm}^{(0)} \eta_{\pm, \vec{n}} = \pm [\phi_{\pm, \vec{n}}, C_{\vec{n}}], \\
Q_{\mp}^{(0)} \eta_{\pm, \vec{n}} &= \mp [\phi_{\pm, \vec{n}}, \phi_{-, \vec{n}}], \\
Q_{\pm}^{(0)} \phi_{\pm, \vec{n}} &= 0, \quad Q_{\mp}^{(0)} \phi_{\pm, \vec{n}} = \mp \eta_{\pm, \vec{n}}
\end{aligned} \tag{51}$$

and

$$\begin{aligned}
\Delta Q_{\pm} \tilde{h}_{x, \vec{n}} &= \frac{\mu}{3} \xi_{\pm x, \vec{n}}, & \Delta Q_{\pm} \tilde{h}_{y, \vec{n}} &= \frac{\mu}{3} \xi_{\pm y, \vec{n}}, & \Delta Q_{\pm} h &= \frac{\mu}{3} \rho_{\pm, \vec{n}}, \\
\Delta Q_{\pm} H_{3, \vec{n}} &= \frac{\mu}{3} \chi_{\pm 3, \vec{n}}, & \Delta Q_{\pm} \eta_{\pm, \vec{n}} &= \frac{2\mu}{3} \phi_{\pm, \vec{n}}, & \Delta Q_{\mp} \eta_{\pm, \vec{n}} &= \pm \frac{\mu}{3} C_{\vec{n}}.
\end{aligned} \tag{52}$$

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